Chapter 32: Interference and Diffraction

Thursday December 1st

- V. IMPORTANT: Final exam will be in HCB103/316
  - HCB316, last names A to J; HCB103, last names K to Z
- Check your exam scores online
- Still 37 unregistered *i*Clickers; send email if unsure.

- Brief review of wave interference (PHY2048)
- Two-slit wave interference
- Diffraction
- Single-slit diffraction

Reading: up to page 575 in the text book (Ch. 32)

Interference of waves

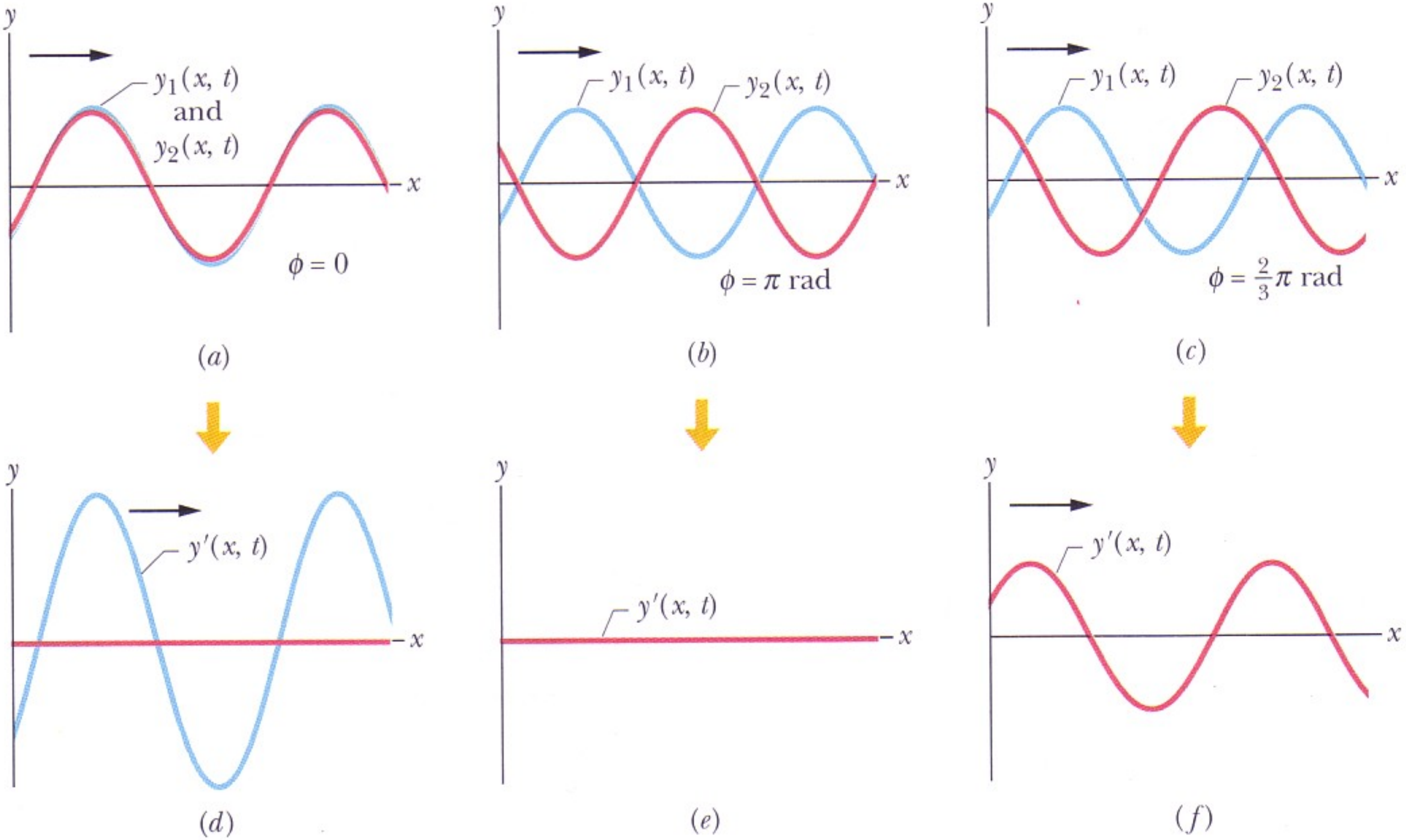
- Suppose two electromagnetic waves with the same frequency, polarization and amplitude travel in the same direction, such that

$$E_1 = E_m \sin\left(kx - \omega t\right)$$

$$E_2 = E_m \sin\left(kx - \omega t + \phi\right)$$

- The waves will add.

Interference of waves



Interference of waves

- Mathematical proof:

$$E_1 = E_m \sin\left(kx - \omega t\right)$$

$$E_2 = E_m \sin\left(kx - \omega t + \phi\right)$$

Then: $$E'\left(x,t\right) = E_1\left(x,t\right) + E_2\left(x,t\right)$$

$$= E_m \sin\left(kx - \omega t\right) + E_m \sin\left(kx - \omega t + \phi\right)$$

But: $$\sin\alpha + \sin\beta = 2\sin\tfrac{1}{2}\left(\alpha + \beta\right)\cos\tfrac{1}{2}\left(\alpha - \beta\right)$$

So: $$E'\left(x,t\right) = \underbrace{\left[2E_m \cos\tfrac{1}{2}\phi\right]}_{\textbf{Amplitude}}\underbrace{\sin\left(kx - \omega t + \tfrac{1}{2}\phi\right)}_{\textbf{Wave part}}$$

Phase shift

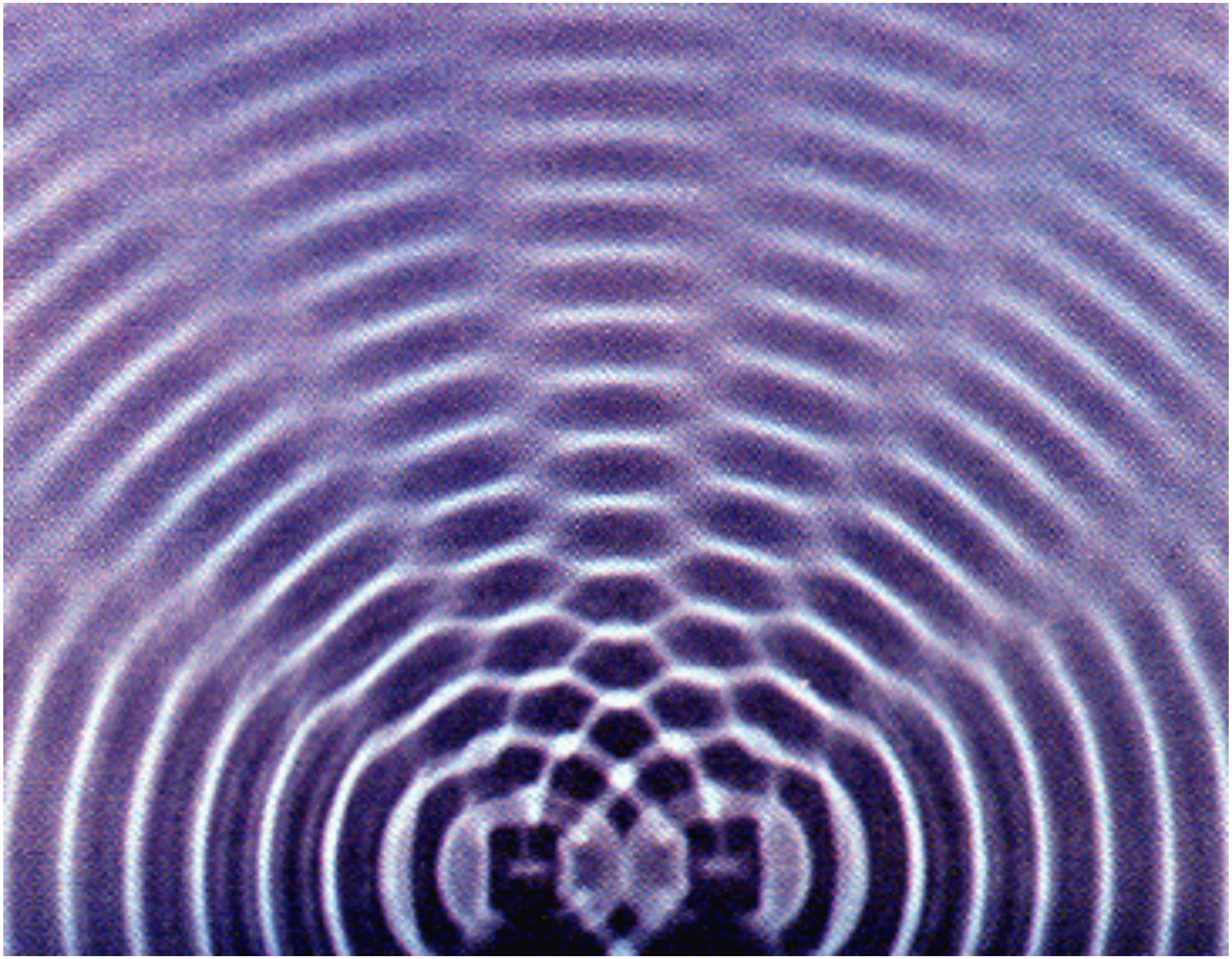
Interference of waves

$$E'(x,t)=\left[2E_m\cos\tfrac{1}{2}\phi\right]\sin\left(kx-\omega t+\tfrac{1}{2}\phi\right)$$

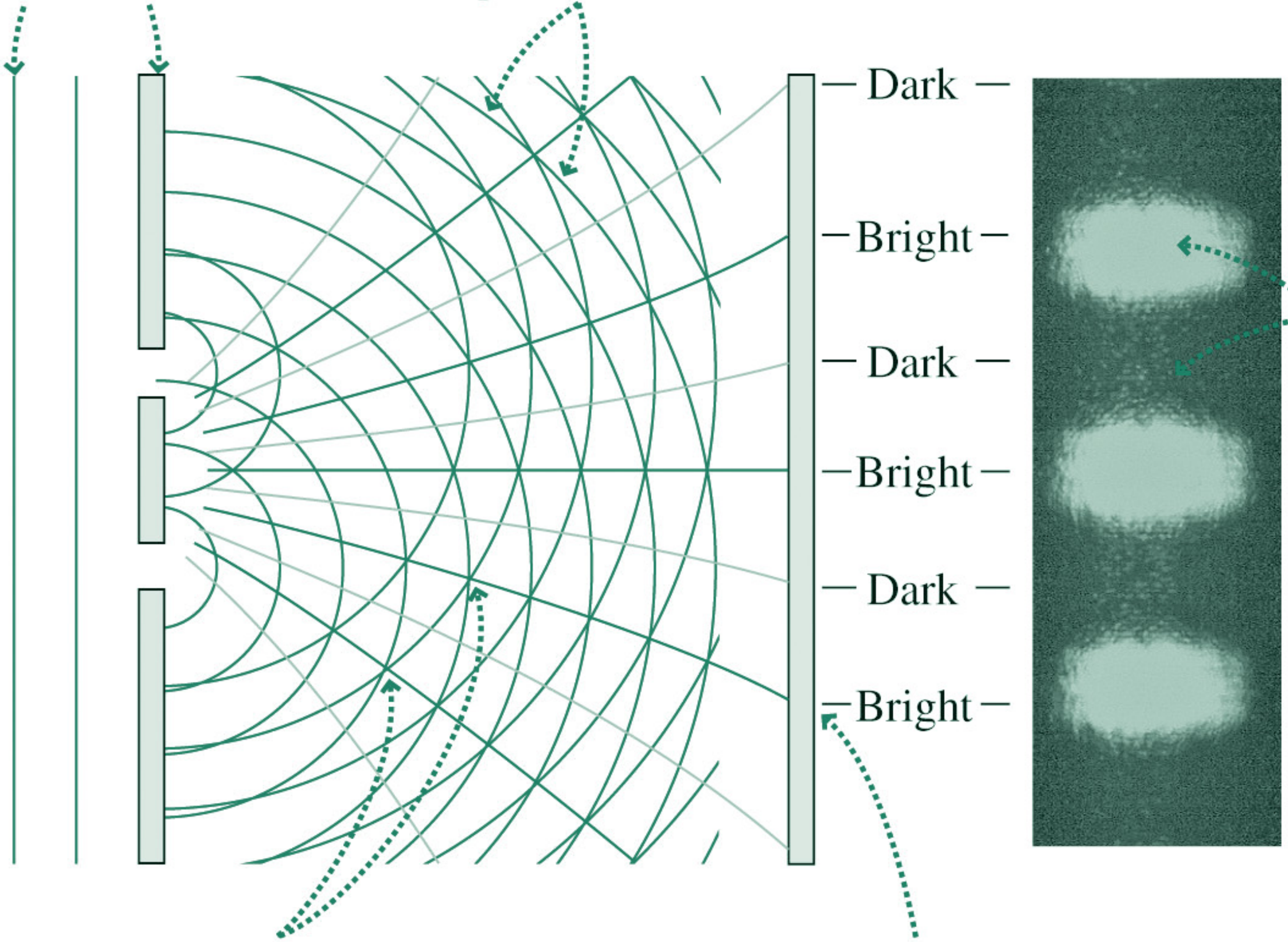
If two electromagnetic waves of the same amplitude, polarization and frequency travel in the **same** direction, they interfere to produce a resultant electromagnetic wave traveling in the same direction.

- If $\varphi = 0$, the waves interfere constructively, $\cos\frac{1}{2}\varphi = 1$ and the wave amplitude is $2E_m$.
- If $\varphi = \pi$, the waves interfere destructively, $\cos(\pi/2) = 0$ and the wave amplitude is 0, *i.e.* no wave at all.
- All other cases are intermediate between an amplitude of 0 and $2E_m$.
- Note that the phase of the resultant wave also depends on the phase difference.

Wave interference - spatial



Double-Slit Interference



Plane waves impinge on barrier with two slits.

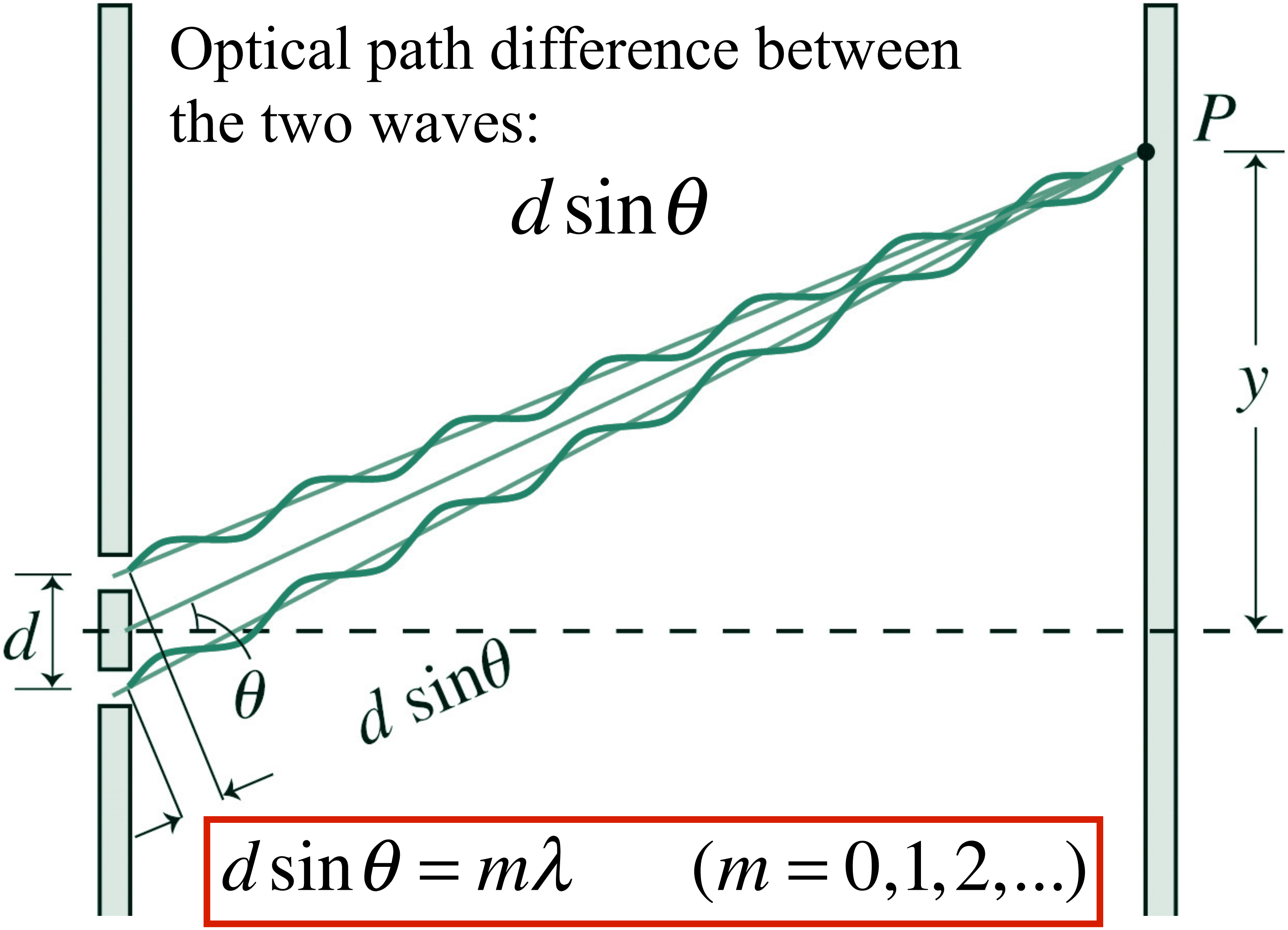
Cylindrical wavefronts spread from each slit.

Photo of an actual interference pattern shows alternating bright and dark fringes.

Along these lines crests meet crests and troughs meet troughs. Thus the waves interfere constructively.

Where lines of constructive interference intersect the screen, bright fringes appear.

Double-Slit Interference



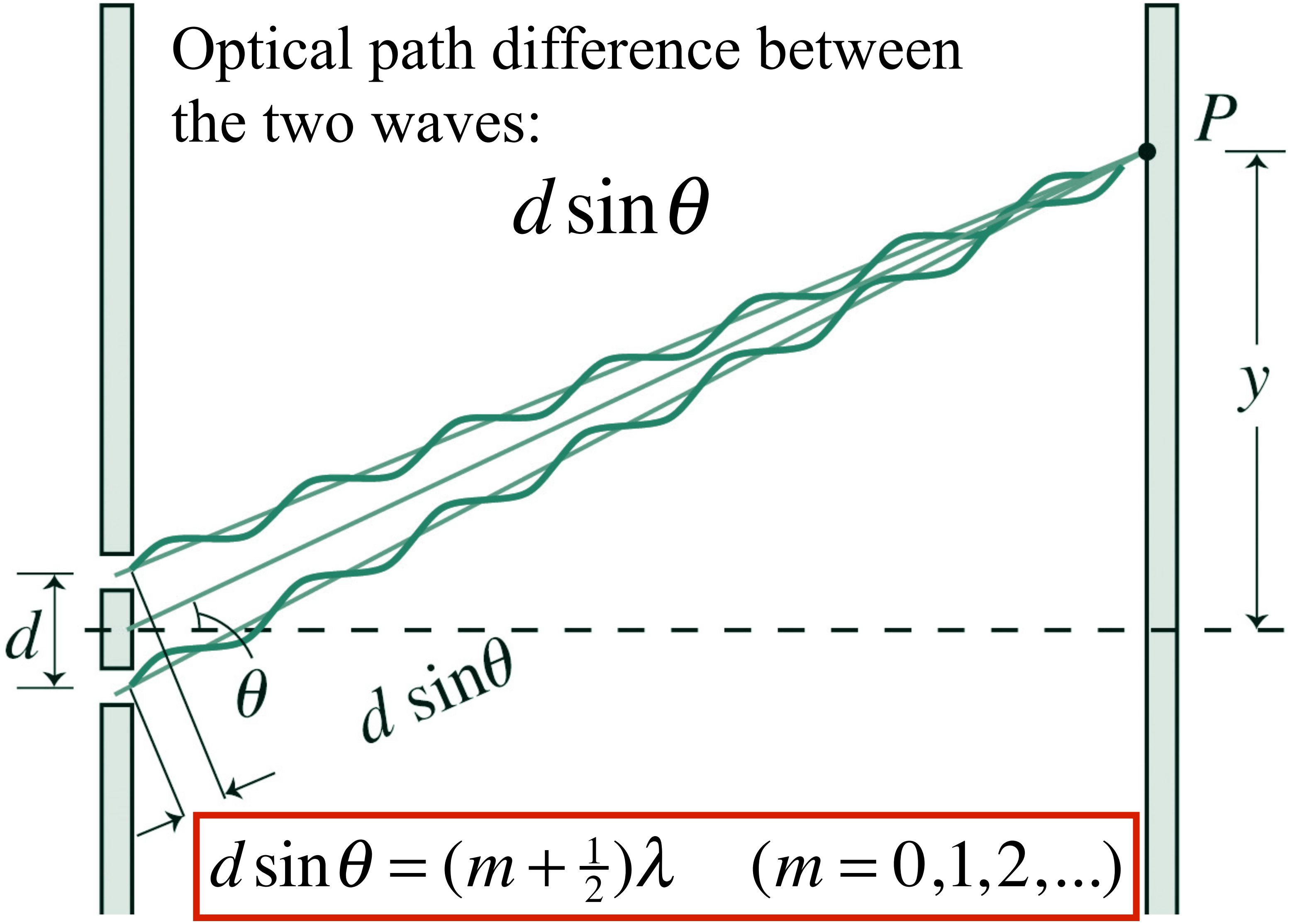
Optical path difference between the two waves:

$$d\sin\theta$$

$$\boxed{d\sin\theta = m\lambda \qquad (m = 0,1,2,\ldots)}$$

When equal to an integer number of wavelengths, constructive interference occurs, i.e., bright fringes.

Double-Slit Interference



Optical path difference between the two waves:

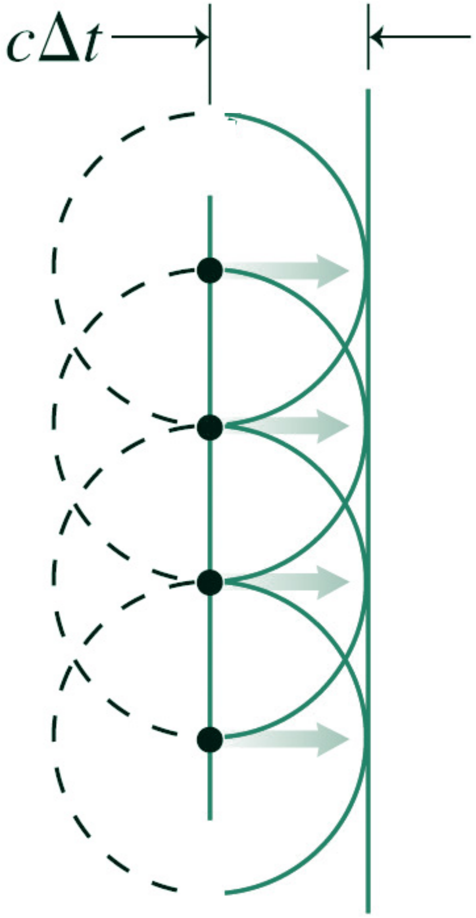
$$d\sin\theta$$

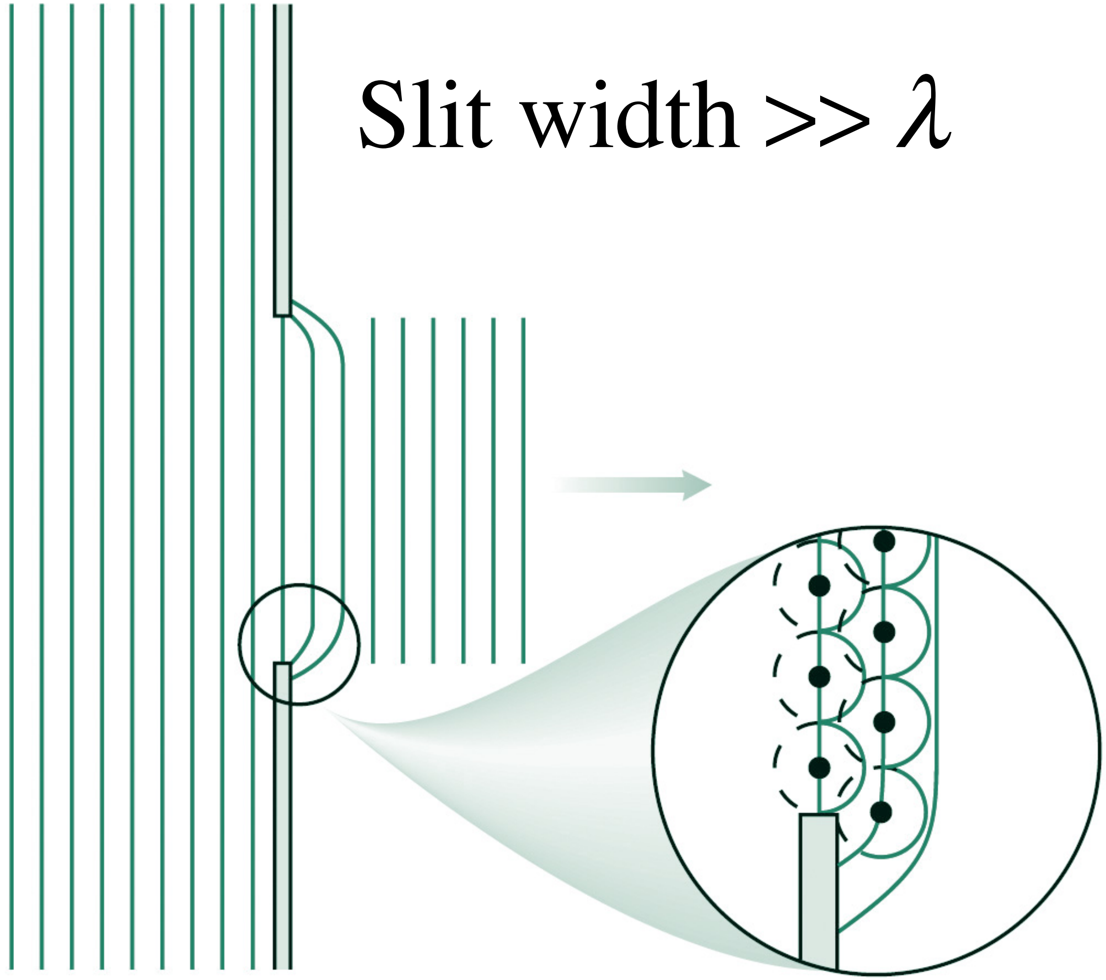
$$\boxed{d\sin\theta = (m+\tfrac{1}{2})\lambda \quad (m = 0,1,2,\ldots)}$$

When equal to a half integer number of wavelengths, destructive interference occurs, i.e., dark fringes.

Diffraction

Huygen's principle: All points on a wavefront act as point sources of spherically propagating wavelets that travel at the speed of light. A short time later, the new wavefront is the unique surface tangent to all of the forward-propagating wavelets.

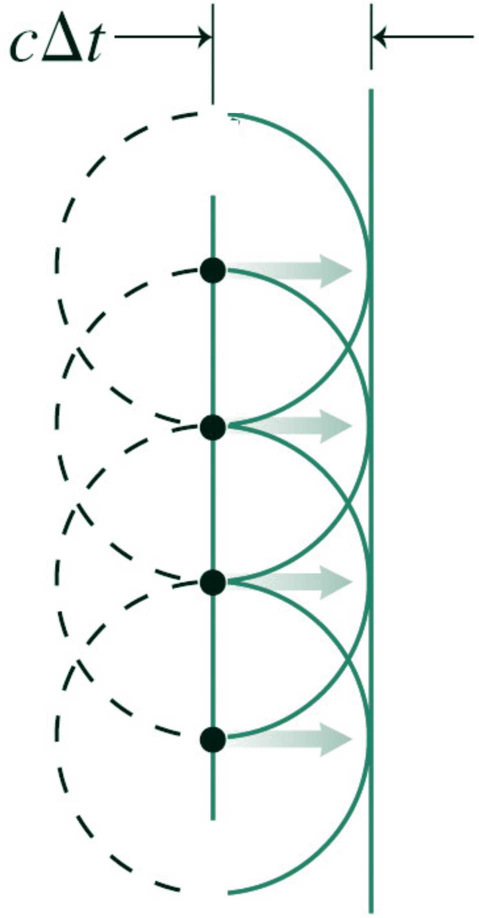




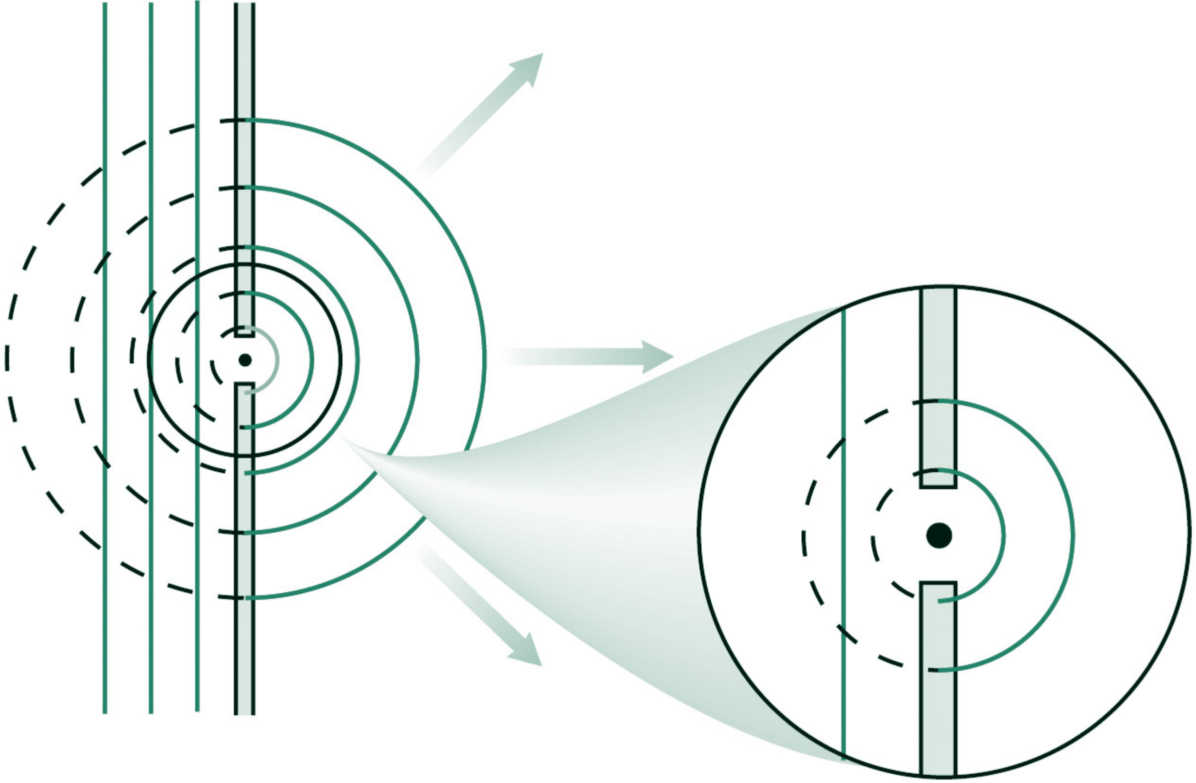
Slit width >> λ

Diffraction

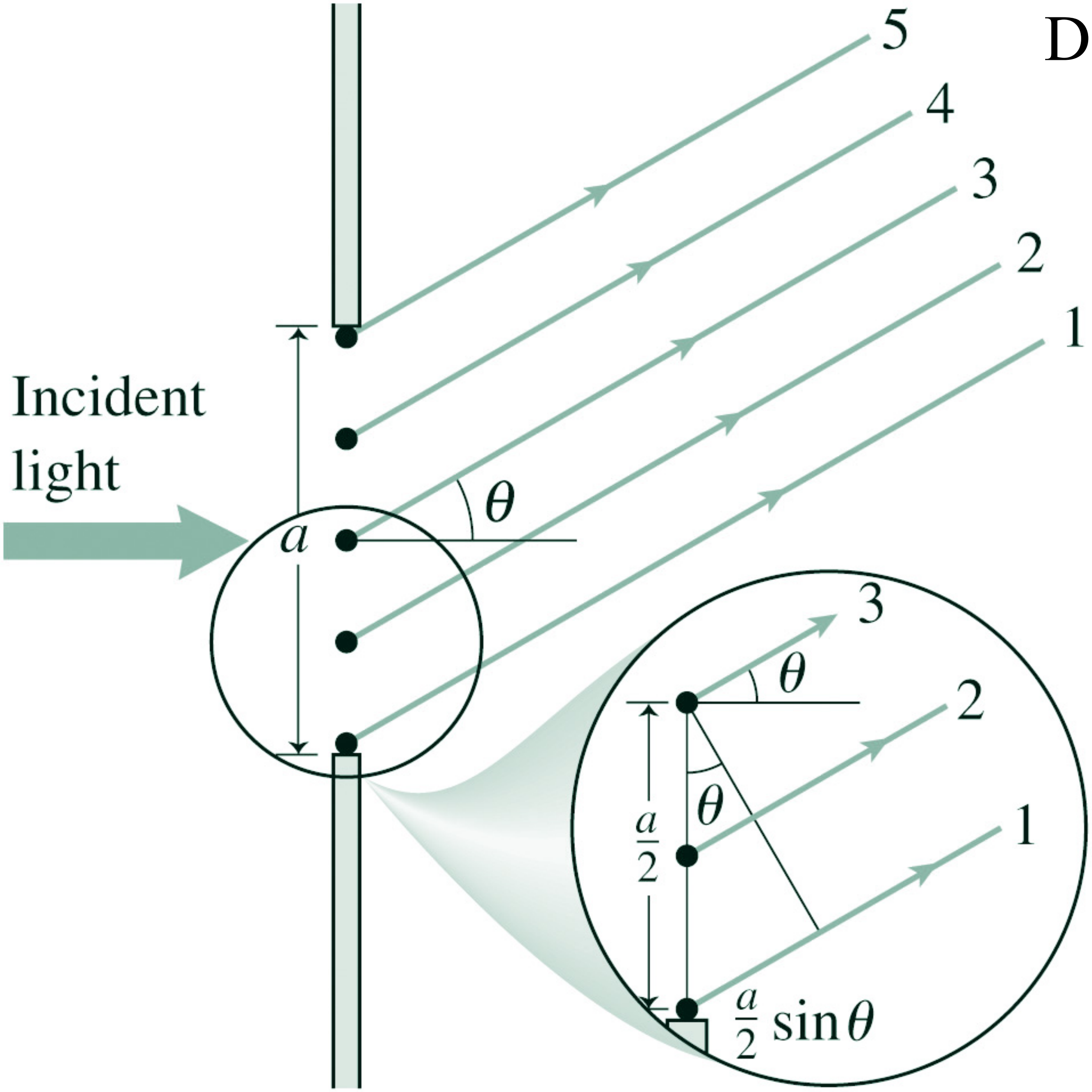
Huygen's principle: All points on a wavefront act as point sources of spherically propagating wavelets that travel at the speed of light. A short time later, the new wavefront is the unique surface tangent to all of the forward-propagating wavelets.



Slit width < λ



Single-Slit Diffraction and Interference



Destructive when:

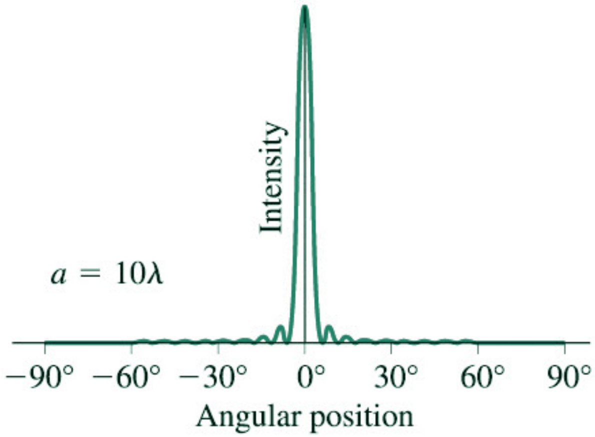
$$\frac{a}{4}\sin\theta = \tfrac{1}{2}\lambda$$

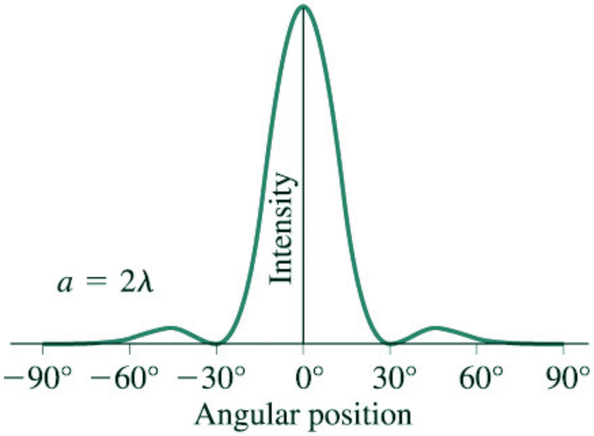
$$\frac{a}{2}\sin\theta = \tfrac{1}{2}\lambda$$

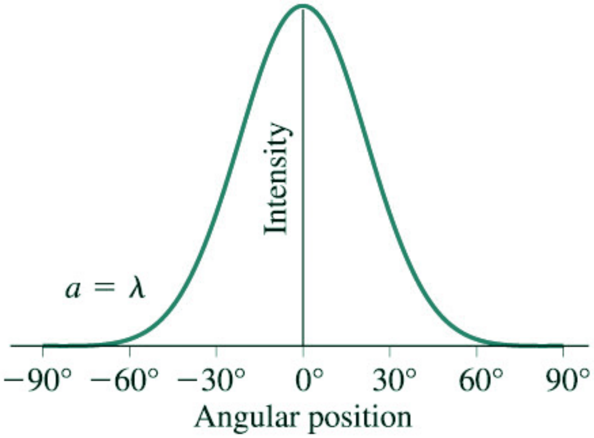
$$\boxed{a\sin\theta = m\lambda}$$

m = integer, but not 0

Single-Slit Diffraction and Interference







Destructive interference when:

$$\boxed{a\sin\theta = m\lambda \qquad (m = 1,2,3,\ldots)}$$

WARNING: This formula looks identical to the two-slit interference formula; **IT IS NOT!!** Note that *a* is the slit width, *d* is the slit separation. Furthermore, the above formula describes destructive interference, whereas the two-slit formula describes constructive interference.